Aeroelasticity, an introduction to fundamental problems - with an historical perspective, examples and homework problems

The four chapters of this text provide an introduction to fundamental static and dynamic aeroelasticity problems using simple idealized models and mathematics to describe essential features of aeroelastic problems. *This text is copyrighted and is intended not for sale or distribution except with the permission of the author.*

Terry A. Weisshaar
Purdue University
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Preface to the Third Edition – My purpose and scope

This book is the result of three decades of experiences teaching a course in aeroelasticity to seniors and first or second year graduate students at Purdue University. During this time I presented lectures and developed homework problems with the objective of introducing students to the complexities of a field in which several major disciplines intersect to produce major problems for high speed flight. These problems affect structural and flight stability problems, re-distribution of external aerodynamic loads and a host of other problems described in the book.

The three decade effort has been challenging, but more than anything, it has been fun. My students have had fun in class and have been contributors to this teaching effort. Some of them have even gone on to professional careers so that they have long ago surpassed my skills to become well-recognized practitioners in this area.

First of all, let me emphasize what this book is not. It is not a book on advanced methods of analysis. It is not a "how to" book describing advanced methods of analysis required to analyze complex configurations. If your job is to compute the flutter speed of a Boeing 787, you will have to acquire that skill elsewhere.

This book focuses on understanding the concepts required to begin to learn about aeroelasticity. It will use simple models and relatively simple mathematics to describe the features of aeroelastic problems. This includes providing an understanding of the mechanisms that create the interactions, what to look for and what to guard against. One of the most important decisions to be made early in a design effort is where to place limited resources. Is aeroelasticity likely to be a problem? Should I do a flutter analysis or wait until later?

An aeroelastician anticipates problems and solves them. Computational procedures have reached the point where such answers are readily obtained. Although computer tools such as NASTRAN have aeroelastic analysis capability, knowing how to input data and request output does not make one an aeroelastician. Putting together models that have the correct boundary conditions and degrees of freedom and interpreting the results is an important task. Requesting and deciphering the plots and fixing problems when they occur is the job of an aeroelastician. That is what this text is all about - understanding the problems of aeroelasticity so that we can understand the fundamental causes and cures for these phenomena.

Finally, I like history and believe that students should understand where they have come from. They should also understand that progress has not come easily and is filled with twists and turns. History is also filled with interesting characters whose contributions have enriched us all. As a result, I have included a small discussion of the background of aeroelastic analysis and listed some of the great contributors to this field. The rest is up to you.

Special thanks are also in order to those who contributed to this effort or inspired me to spend most of my career in this area. I am also indebted to AAE556 students Janene Silvers, Jacob Haderlie, Matt Snyder, Satadru Roy and Ben Goldman, who read the first three Chapters of the manuscript diligently and provided comments.

Terry A. Weisshaar
West Linn, Oregon

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About the author …

Professor Weisshaar is Professor Emeritus, Purdue University, West Lafayette, Indiana, retiring from Purdue in May 2011 after a 45 year career that included teaching and research at three major universities, service on major governmental and industrial advisory panels. He currently resides in West Linn, Oregon, near Portland. He is an internationally recognized authority in the areas of aeroelasticity, smart adaptive structures and aircraft design. He is recognized as a visionary in diverse areas of aerospace systems design and component technologies. His national awards include

- August 2006 he received the Secretary of Defense Outstanding Achievement Award to recognize his efforts in developing morphing air vehicles for the U.S. military. The citation read, in part, *"He identified and aggressively pursued high-leverage opportunities and programs which dramatically improve our warfighting capability in key areas. Dr. Weisshaar produced the first integrated technology aircraft designs for a unique bio-inspired air vehicle. This new vehicle is emblematic of a new class of multifunctional vehicles with capabilities for multiple military roles. The distinctive accomplishments of Dr. Weisshaar reflect great credit upon himself, DARPA, and the Office of the Secretary of Defense."*

Dr. Weisshaar (left) receiving the DoD Secretary of Defense, Distinguished Service Award, 2006

- April 2005 he received the AIAA Structural Dynamics and Materials Award to recognize *"fundamental contributions to research and education in aeroelastic design, aeroelastic tailoring, and the application of smart advanced composites."*
- October 1998 he received the Air Force Exceptional Civilian Service Decoration in 1998, an award to recognize service and accomplishments while on the U.S. Air Force Scientific Advisory Board. The citation read, in part *"… your quality reviews, both as a member and as a Chair of the Materials Panel, of the Air Force science and technology investment portfolio aided immeasurably in identifying key investments needed to lead to revolutionary breakthroughs in the 21st century. Your efforts have defined the future technology path for the Air Force. Your unique perspectives and outstanding accomplishments reflect the highest credit upon yourself and the United States Air Force."*

2005 AIAA SDM Award Recipient (on right)

- May 1993 he was elected as a Fellow of the American Institute of Aeronautics and Astronautics (AIAA) which recognized his *"...significant contributions to research, education and service to the aeronautics and astronautics community, specifically in the areas of aeroelasticity and aeroelastic tailoring and for his pioneering research that established the fundamentals of aeroelastic interaction with advanced composites."*
- 1983, he was the co-recipient of the ASME Structures and Materials Award, which recognized a research contribution to forward swept wing aeroelasticity.

Dr. Weisshaar is a graduate of Northwestern University (1965, B.S.ME), Massachusetts Institute of Technology (1966, S.M.AE) and Stanford University (Ph.D., 1971).

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Table of contents

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CHAPTER ONE - An Introduction to Aeroelasticity

1.0 - Background

Two factors drive aviation development: 1) the quest for speed; and, 2) the competition for new air vehicle military and commercial applications. These factors trigger the appearance of new aircraft shapes, devices and materials, as well as applications of new technologies such as avionics. These factors have created and continue to create new challenges for the engineering discipline known as *aeroelasticity*.

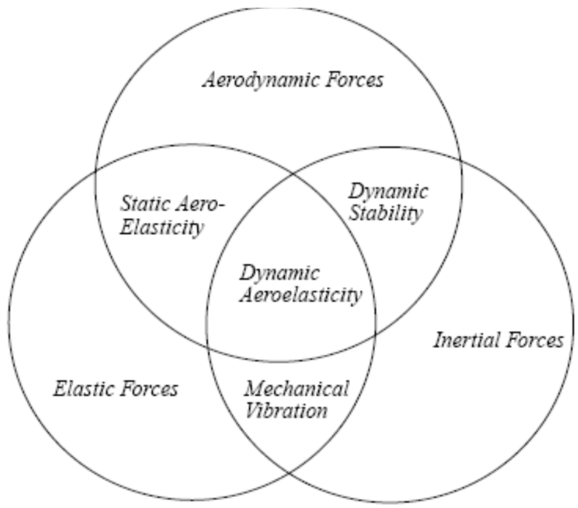

Figure 1.1 - Three-ring aeroelastic interaction Venn diagram.

This chapter introduces the field of aeroelasticity and shows how this discipline has been and remains important to aerospace development. This introduction reviews the part that aeroelasticity has played in aeronautical development during the past century. The development of Aeroelasticity developments have been reactionary; the appearance of aeroelastic phenomena has largely been unanticipated and expensive in terms of resources required to fix problems, projects delayed or cancelled and, most tragically, lives lost as the result of accidents that can be directly traced to aeroelasticity. Understanding the causes and solutions to problems and anticipation of these problems are the necessary first steps to any engineering endeavor, even before the first equation is written. At the end of this chapter you should have a clear idea about the scope of aeroelastic problems and their origin.

Figure 1.2 – Boeing 767 static structural proof test – loading to wing failure produces considerable wing deformation

Aeroelasticity studies, analyzes and harnesses the interactions among aerodynamic forces, structural deformation (elasticity) and motion (dynamics) of aerodynamic and hydrodynamic lifting surfaces. The term *aeroelasticity* was coined in the early 1930's by Alfred Pugsley and Harold Roxbee Cox, two engineers at the British Royal Aircraft Establishment (RAE) whose studies advanced the knowledge

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and importance of aeroelasticity, its consequences and its cures. The interaction of these interdisciplinary activities is summarized in the "three-ring" Venn diagram shown in Figure 1.1. Overlaps among the three areas define phenomena that we will discover and discuss in the next three chapters.

Figure 1.3 – NASA Pathfinder UAV

Aeroelastic interactions determine airplane loads and influence flight performance in four primary areas: 1) wing and tail surface lift redistribution that change external loads from preliminary loads computed on rigid surfaces; 2) stability derivatives, including lift effectiveness, that affects flight static and dynamic control features such as aircraft trim and dynamic response; 3) control effectiveness, including aileron reversal, that limits maneuverability; 4) aircraft structural dynamic response to atmospheric turbulence and buffeting, as well as structural stability, in particular flutter.

For most structures such as cars and buildings, noticeable structural deformation is objectionable and disconcerting. On the other hand, efficient wing and tail surface structural design permits substantial structural bending and twisting deformation during flight, as indicated in the picture of the Boeing 767 ground test shown in Figure 1.2 and the flight test picture of the NASA Pathfinder UAV in Figure 1.3. Wing twist produces changes in local angle of attack on swept and unswept surfaces. This twist produces angles of attack along the wing that also produces substantial wing aerodynamic loads. Sweptback wing bending reduces local angle of attack, while sweptforward wing bending increases local angle of attack. These are aeroelastic interactions and may be either static or dynamic.

To describe aeroelastic phenomena, let's return to the beginning of manned powered flight and use aircraft development history to introduce aeroelastic phenomena. Two excellent survey articles[1, 2] cover a variety of early historical events related to aeroelasticity. We will draw heavily on these articles, supplementing them with our own examples.

Aeroelastic loads created by lifting surface distortion have been an important part of aeronautical engineering from the very beginning of controlled, powered flight. In the late 1890's and early 1900's, Professor Samuel P. Langley developed an airplane, the *Aerodrome*, capable of being launched from a houseboat anchored in the Potomac River near Washington, D.C. This airplane failed on each of two attempts in late 1903.

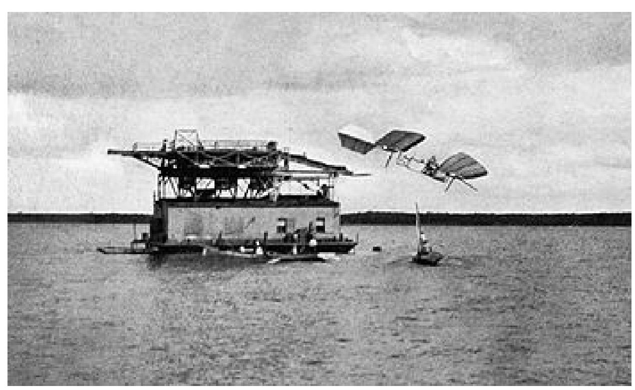

Figure 1.4 - Langley *Aerodrome* failure, October 7, 1903.

The first failure, on October 7, 1903, (shown in the photograph in Figure 1.4) was probably due to a front-wing guy post catching on the

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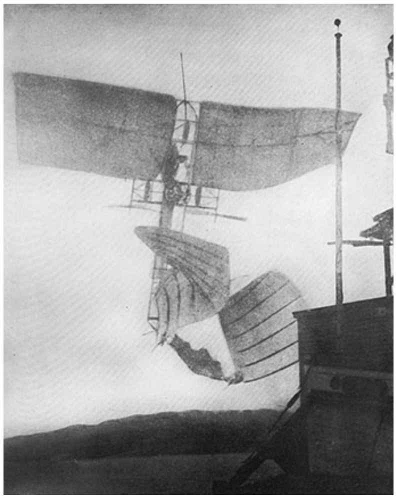

Figure 1.5 – Langley Aerodrome aeroelastic launch failure December 1903 showing forward wing collapse

catapult launch mechanism. This failure produced a sharp criticism from a *New York Times* editor who wrote, on October 9, 1903, "*it might be assumed that the flying machine which will really fly might be evolved by the combined and continuous efforts of mathematicians and mechanicians in from one million to ten million years…*"

The failure of the second (and final) *Aerodrome* flight is shown in Figure 1.5. For a number of years this failure was attributed to insufficient wing-torsional stiffness that led to structural static divergence, an instability much like beam/column buckling that leads to excessive torsional deformation of the wing. We now know, as the result of testing on the original Langley machine, that divergence was not the cause of failure (see Reference 2). However, legends die hard.

On October 7, 1903, the day that Langley's first *Aerodrome* flight failed, the Wright Brothers recorded in their diary, *"We began unpacking today."* In 1901 the Wright Brothers used a tethered kite, sketched in Figure 1.6, to demonstrate *wing warping*. Wing warping uses controlled, anti-symmetrical bi-plane wing structural twisting displacement to create aerodynamic rolling moments.

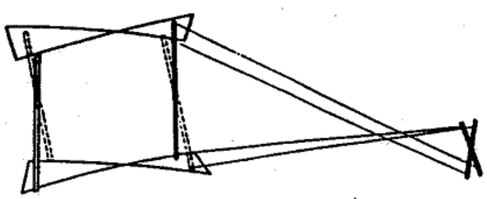

Figure 1.6 – 1901 Wright Brothers wing warping kite.

The wing warping concept had been tested by Edson Gallaudet as early as 1898. He did not pursue, publish or patent this idea. The Wright Brothers, unaware of Gallaudet's work, obtained a patent for wing warping control in 1905, creating a financial boon for themselves an regarded all aircraft control as being covered by this patent, retarding aeronautical design progress as a result. Warping depends on building torsionally flexible wing surfaces easily distorted by the pilot, but the wings are also easily distorted by the airstream that may produce self-excited, unintended airloads.

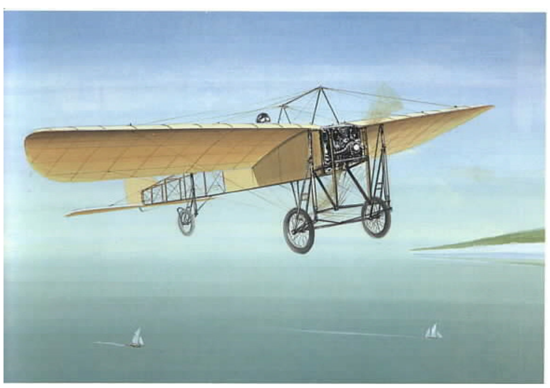

Figure 1.7 – Louis Bleriot crossing the English Channel from France. He captured the London Daily Mail prize of 1000 pounds Sterling.

Most early monoplanes used wing warping roll control. These included the Bleriot XI, shown in Figures 1.7 and 1.8, the British Bristol Prier Monoplane (Figure 1.9), and, as late as 1915, the Fokker Eindecker.

On July 25, 1909 Louis Bleriot flew across the English Channel from France at a speed of about 60 mph. His Bleriot XI was an externally braced monoplane with wing warping control. This design was immediately popular. After the flight it was

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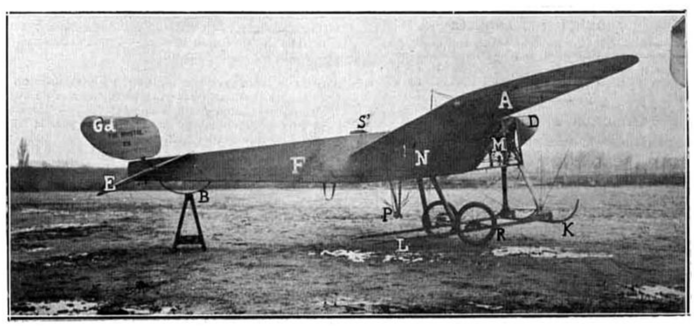
apparent that England was only a few minutes from the European Continent; the British Navy no longer gave the security it had for hundreds of years.

Figure 1.8 – The wire-braced Bleriot XI with a wing warping control system was flown by Harriet Quimby between Dover and Calais on April 16, 1912. She was America's first licensed female pilot, but was killed a few months later in Boston, Massachusetts in a Bleriot monoplane.

During the first decade of powered flight, airplane speeds were low enough and structural stiffness large enough that loads due to airload induced wing and tail twisting deformation were inconsequential with a few spectacular exceptions.

Like the Wright Brothers Flyer, Bleriot's wing warping roll control required relatively low wing torsional stiffness. As engine power and airspeed increased, low torsional stiffness created aeroelastic problems that led to wing failures at high speeds.

Both the *Bleriot XI* and the *British Bristol Prier* (Figure 1.9) were involved in fatal accidents in England in 1912. At first, the structural failures were thought to be the result of insufficient wire bracing strength. Bleriot strengthened the Bleriot XI guy wires and increased the main wing spar size, but wing failures still occurred. Bleriot had inadvertently created a new aeroelastic effect, later called *wing divergence*, discussed in Chapter 2. At the time nothing was known about such problems so that the load-deformation interaction mechanism was not recognized.



LE MONOPLAN " BRISTOL " ; VUE LATÉRALE. — Pour la signification des lettres voir légende des dessins ci-après (*Photo Branger*)

Figure 1.9 - Bristol Prier monoplane – top speed 68 mph, first flight 1911 (published in *L'Aérophile* 1 June 1912, page 242)

On September 12, 1912 the British War Office banned Royal Flying Corps pilots from flying monoplanes after a series of accidents involving the Bristol Prier monoplane.[3] Some witnesses to these crashes reported in-flight explosions. France issued its own monoplane flight ban soon after. These temporary bans created an anti-monoplane bias lasting through World War I. As a result bi-plane designs remained supreme for another decade.

Almost 15 years later, in 1926, Hans Reissner published, in Germany, his landmark paper *"Neuer Probleme aus der Flugzeugstatik"* (*New Static Structural Problems of Wings).*[4] This paper provided a clear mathematical and physical understanding of the origin of static aeroelastic phenomenon such as lift effectiveness and wing divergence. Similar research papers and reports soon appeared in other countries.

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1.1 - Aircraft structural dynamic response, stability and flutter

During World War I, a self-excited, vibratory aeroelastic instability, later called *flutter,* occurred on the horizontal tail of the British Handley Page O/400 bi-plane bomber shown in Figure 1.10. ***Flutter is a dynamic, oscillatory structural instability enabled by interactions between unsteady aerodynamic forces and moments created by vibratory motion of lifting surfaces and the vehicles to which these surfaces are attached.*** About ten years later, the term "flutter" first appeared in 1924 in a published document by R.T. Glazebrook in the *Yearbook of the British Aeronautical Committee.*

Figure 1.10 – British Handley Page O/400 bomber. Its 100 foot wings folded back to allow it to use existing hangers

Investigations in 1916 revealed that the O/400 tail flutter failure was caused by interaction between O/400 fuselage twisting oscillation and the anti-symmetrical pitch rotations of the independently actuated right and left elevators. This vibration coupling was eliminated by connecting the O/400 elevators to a common torque tube to eliminate anti-symmetrical elevator motion. The attachment of both elevators to the same torque tube became standard design practice.

Figure 1.11 – Farman III showing the first ailerons

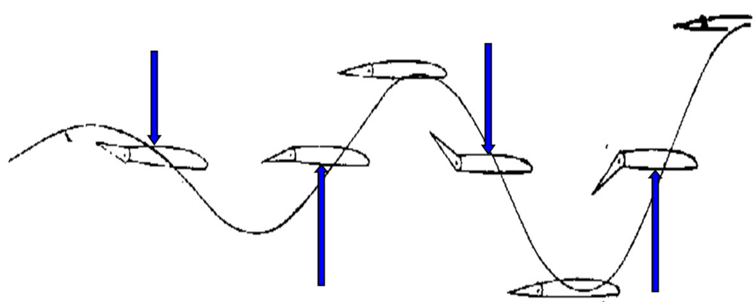

Figure 1.12 – Interactions between ailerons and wing bending motions at high speed produced the first catastrophic flutter events in the 1920's. Flutter requires two different types of structural vibrations, one to generate aerodynamic forces, the other to produce deflections through which these forces do work (Chapter 4).

Henri Farman equipped his airplane, the Farman III (shown in 1.11), with four flap-like ailerons fitted at the outboard trailing edges of both the upper and lower wings. Farman was the first to make ailerons an integral part of the wing. Farman's aileron was more effective and less complicated than wing warping; it was quickly adopted by aircraft designers. Although Orville Wright held on to wing warping, in 1915 he finally was converted to using ailerons on his aircraft.

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Following World War I, engines continued to become more powerful and horsepower-to-weight ratios increased. As airspeeds increased, monoplane designs reappeared, this time with new low drag, structurally stiff, semi-monocoque wing designs. A new type of aeroelastic instability, called *wing-aileron flutter* plagued aircraft designs. Just as the wing warping type of control had led to wing divergence, the new aileron control surfaces led to dynamic aeroelastic failures.

As shown in Figure 1.12, wing-aileron flutter is a self-excited vibration that occurs when lift generated by the oscillation of an aileron creates wing bending or torsion deformation. The oscillation frequency depends on airspeed because the aileron acts like a weathervane; its rotational stiffness and natural frequency increase as airspeed increases. The aileron acceleration, as well as the airloads transmitted to the wing, force the wing oscillations and create interactive coupled vibration.

Flutter belongs to a special class of mechanics problems called *non-conservative* problems. The flutter mechanism depends on flying at or above an airspeed and altitude to allow two or more aircraft vibration modes to interact or *couple* together. Flutter is categorized into at least five different areas, each with its own characteristic modes of motion: 1) classical flutter – wing bending & torsion; 2) control surface flutter – surface rotation and wing bending; 3) empennage flutter – fuselage torsion and tail torsion; 4) stall flutter – wing torsion; 5) body freedom flutter – wing bending and fuselage pitch.

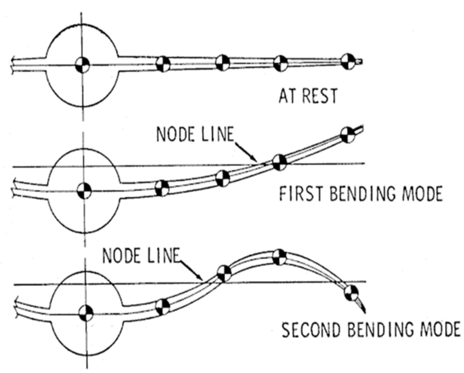

Figure 1.13 – Wing vibration mode shapes

Aircraft and missile resonant natural frequencies, such as those shown in Figure 1.13, depend on stiffness, mass distribution, airspeed, altitude and Mach number. Figure 1.14 shows time histories of three possible types of dynamic response at a point on a wing surface responding to a disturbance during flight at different airspeeds.

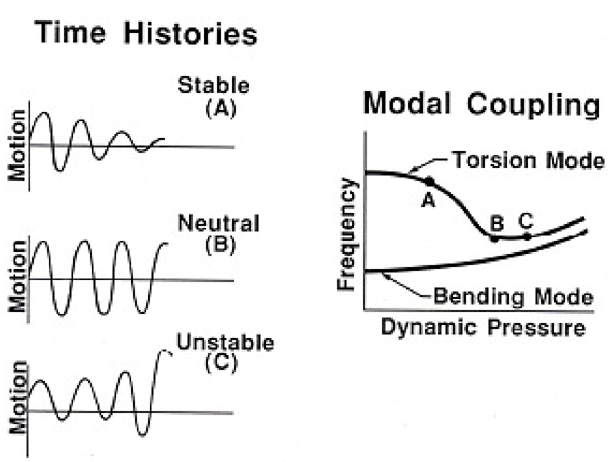

Figure 1.14 – Aircraft vibration modes couple together to allow airstream energy to be absorbed by the structure.

Disturbances decay with time at an airspeed corresponding to *point A* where the resonant natural frequencies are well separated. As airspeed is increased to point B, an initial disturbance produces (after some transient motion) harmonic oscillatory motion at a fixed amplitude. An attempt to operate at the airspeed associated with point C will lead to disaster, since the amplitude of the response to the initial disturbance grows rapidly with time. The motions associated with airspeeds at A, B, C are classified as stable, neutrally stable and unstable, respectively.

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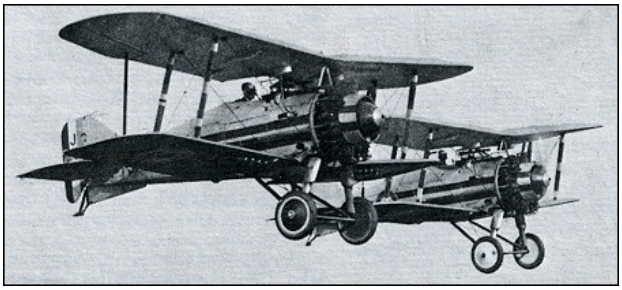

Figure 1.15 – British Gloster-Grebe aircraft, wing-aileron flutter victim, ca. 1923. Top speed, 151 mph.

Flutter *is not forced resonant response*. The airstream causing flutter is steady and non-oscillatory until the system is disturbed. In addition, without internal structural damping, resonance response amplitude grows linearly with time, while the flutter dynamic response has an exponential increase until the structure is destroyed or some nonlinear mechanism limits the response amplitude.

Figure 1.16 – British Gloster Gamecock, successor to the Grebe. With a top speed of 150 mph, this airplane had vertical fin/rudder flutter problems and a high accident rate. Note the large rudder surface.

Wing/aileron flutter and its counterpart, rudder/vertical tail flutter, were all too common in the 1920's. In 1923, the then-new British Gloster-Grebe bi-plane fighter, shown in Figure 1.15, encountered wing/aileron flutter. The Grebe's problem was fixed by adding wing struts to stiffen the wing. In 1925 the last wooden British fighter, the Gloster Gamecock (shown in Figure 1.16) encountered rudder/vertical tail flutter. This was also eliminated by mass balancing the rudder to decouple vibration modes.

1.2 - Aeroelastic analysis comes of age in Britain, Russia and Germany

Idealized models play an essential role in technology development. In the beginning, relatively low fidelity models are required to define and understand the interaction of fundamental parameters related to the phenomena. Later, increased fidelity is required to describe or simulate these interactions for actual designs. In aeroelasticity the essential equations must contain descriptions of structural, aerodynamic and inertial features. While all three of these model components are important, none is more important than the theoretical aerodynamic model, particularly the model that describes transient or unsteady aerodynamic loads that develop in response to structural deformation. Anderson's *A History of Aerodynamics* is an excellent reference describing the birth and development of aerodynamic theory. [5]

Early airplane designers and constructors relied on craftsmanship with little or no analysis. Rudimentary testing was used to confirm their choices of structural sizing with flight testing left to confirm aerodynamic choices. Little if any aerodynamic theory was used in the design of aircraft, even up to and including World War I.

Loftin[6] notes *"Aircraft design during World War I was more inventive, intuitive and daring than anything else… The principles of aerodynamics that form so important a part of aircraft design today were relatively little understood by aircraft designers during the War… In the area of engineering in*

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which structural strength, lightweight and aerodynamic efficiency are so important, it is indeed surprising that a number of relatively good aircraft were produced."

Frederick Lanchester published *Aerodynamics* in 1907.[7] This book contained a compendium of ideas he had worked on since the early 1890's. He was the original source of the circulation theory of aerodynamic lift and described the vortex action required to create aerodynamic lift. This theory was rejected by scientific journals. Anderson's history notes that Lanchester's work suffered from two problems. First of all, Lanchester was not a clear writer and second, he was not quantitative; he did not provide a way to compute lift and drag. The quantitative contributions were provided by independent researchers such as Wilhelm Kutta who had no knowledge of Lanchester's efforts.

Wilhelm Kutta's aerodynamic lift calculations were first published in July 1902 in Germany in a short note entitled "Lifting Forces in Flowing Fluids." This was the first publication to describe how to compute the lift on a wing with an infinite wing span, in incompressible flow. Kutta's work did not use Lanchester's circulation theory explicitly. Kutta re-visited his theoretical work in 1910 and developed the relationship between lift, density, velocity and circulation.

In 1906, Nikolai Joukowski (1847-1921), a professor in Moscow, published two technical notes in obscure journals. He derived the classical formula $L=\rho V\Gamma$ relating lift per unit span to density, ρ, velocity, V, and circulation strength, Γ. These developments created the Kutta-Joukowski theory of lift, the foundation of aerodynamic theory for almost forty years.

These developments were only the tip of the iceberg for theoretical development. In Reference 5, Anderson notes that *"…research in aeronautics could no longer be left to misguided dreamers and madmen; once aeronautical work became respectable , that opened the floodgates to a whole new world of research problems, to which twentieth century academicians flocked."*

The Germans were the first to develop special comprehensive laboratories incorporating applied/experimental and theoretical aerodynamics. In fact, the most important engineering development from circulation theory was lifting line theory developed by Ludwig Prandtl (1875-1953) in Göttingen, Germany. This theory, developed during World War I, predicted lift and induced drag for finite span wings. Prandtl was a giant in his field, a man who comes along only once every few decades.

Prandtl's work, plus the inspiration and the ideas he provided to others, including Theodore von Karman, contributed greatly to aeronautical research. His efforts at the University of Göttingen produced outstanding researchers and research, both theoretical and applied. This operation and spin-offs were the center of serious aeronautical development for two decades after World War I. In addition to the work in theoretical aerodynamics in steady flow, Prandtl's group at the University of Göttingen produced seminal research in unsteady aerodynamic theory.

In 1922, Prandtl's student Walter Birnbaum published his thesis "*The two-dimensional problem of the flapping wing.*"[8] This thesis was an outgrowth of Prandtl's interest in aeroelastic oscillations. For the first time, wing flutter was described as a structural dynamic stability problem. He considered an airfoil which, during flight "carried out a periodic up and down movement, where, simultaneously, the angle of attack may be changed by a rotation of an airfoil" (This was the first time that a "typical

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section" was used to analyze flutter). He also used the term *reduced frequency* $k=\omega c/V$ as a key parameter to define the unsteadiness of the flow. Reduced frequency combines the oscillation frequency ω, the airfoil chord c and the airspeed, V. The typical section model will be described and used in Chapters 2, 3 and 4. Prantdl described the flutter analysis problem at a meeting in Innsbruck, Austria in September 1922.

Birnbaum's theoretical model used a flat plate-like airfoil oscillating harmonically at constant frequency ω with both rotational (twisting) and "flapping" (also called later "plunge" or bending) freedoms. This led to an integral equation that he solved for reduced frequencies up to k=0.12. He showed that the closer the center of gravity and the shear center were to the leading edge, the higher the flutter speed. He also showed that proximity between the fundamental bending frequency and torsion frequency promoted low flutter speeds. Birnbaum also showed that wing torsional divergence was an aeroelastic instability problem involving interaction between aerodynamic forces and wing torsional deformation.[9]

Two years later, in 1924, Herbert Wagner, another protégé of Prandtl, developed the Wagner function, the first transient solution describing theoretical lift development on a two-dimensional flat plate airfoil when it is given an instantaneous increase in its angle of attack. We will discuss the Wagner function in Chapter 4.

In the Netherlands, in 1922, Albert von Baumhauer (1891-1939) and C. Köning identified the cause of and solutions to wing/aileron flutter by conducting experimental and theoretical investigations of wing/aileron dynamic interaction on the Van Berkel W. B. seaplane. [10] Their conclusion was that *"… the wing could perform violent oscillations. A theoretical and experimental investigation led to the conclusion, that in some cases an unstable oscillation of the wing-aileron system under the influence of the elastic and aerodynamic forces is possible without further external causes."*

Von Baumhauer and Köning were the first to understand that flutter can be prevented by decoupling modes of vibration by attaching small weights ahead of the aileron hinge line; this mass balancing solution is still used today. Their research, originally published in 1922, and 1923, was re-published as a NACA technical memorandum. [11] Vom Baumhauer later pioneered Dutch work in helicopters and the aerodynamics of windmills. He died in a Boeing Stratoliner plane crash near Seattle in March 1939 while serving as a representative of the Netherlands Airworthiness Board.[12] Köning went on to be the laboratory director of the Dutch National Aeronautical Laboratory.

At the British Royal Aeronautical Establishment at Farnborough, British engineers were also busy formulating theories to aid understanding of aeroelastic phenomena and to help designers avoid them. By the end of the 1920's the fundamentals of flutter were clearly understood.[13]

The value of the contributions to aeroelasticity generated by researchers at the Royal Aeronautical Establishment at Farnborough cannot be over-estimated. While German contributors such as Prantdl and Wagner in Göttingen excelled in aerodynamics and unsteady aerodynamics research, British researchers such as A.G. Pugsley, A.R. Collar and A.R. Cox unraveled the mysteries of aircraft roll effectiveness and flutter in the 1920's and 1930's.

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In the United States, Theodorsen's work at the NACA on unsteady aerodynamics added further fidelity to unsteady aeroelastic analysis of the *typical section*.[14] The key parameter in Theodorsen's calculations is the reduced frequency of oscillation, *k* although Theodorsen wrote k as $k=\omega b/V$, where b is the wing section semi-chord while V is the airspeed and ω is the frequency of the airfoil oscillation. Flow unsteadiness is very important for calculations for systems with reduced frequencies beginning at about $k = {}^{\omega b}\!/_{V} \approx 0.04$. In 1942 Smilg and Wasserman, engineers at the U.S. Army's Wright Field in Dayton, Ohio outlined a flutter analysis procedure called the *V-g method*.[15] This method was a standard computational technique for several decades; we will describe this analytical method in Chapter 4.

1.3- Control effectiveness

In addition to aeroelastic stability, other aeroelastic response phenomena plagued aircraft in the 1920's. ***Control effectiveness*** is the ability of a control surface such as an aileron or a rudder to produce aerodynamic forces and moments to control airplane orientation and maneuver along a flight path. Asymmetrical aileron rotation produces rolling acceleration and roll rate. The ability to create a *terminal* or *steady-state roll rate* is the primary measure of aileron effectiveness.

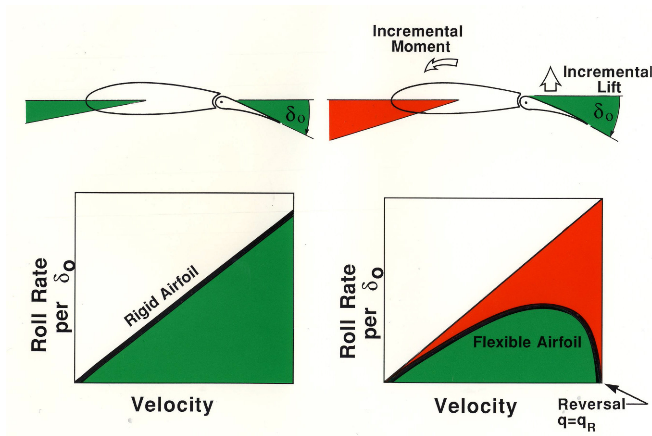

Figure 1.17 – Control effectiveness declines with increasing airspeed

Consider Figure 1.17. Without wing torsional flexibility, the terminal roll rate is a linear function of airspeed. Rotating the aileron downward produces an effective angle of attack to produce lift, but also twists the wing surface nose-down, reducing the local wing angle of attack and reducing lift that creates the rolling moment. The size of the nose-down twisting moment and nose-down twist depends upon the: 1) size of the control surface; 2) amount of aileron deflection; 3) structural stiffness; and, 4) dynamic pressure, *q*.

As indicated in Figure 1.17, the terminal roll rate becomes maximum and then declines rapidly as airspeed increases. At a special airspeed, called the *aileron reversal speed,* the ailerons will not generate a rolling moment even though there is substantial wing surface distortion and substantial aileron rotation. At speeds above the reversal speed, the aileron produces a roll rate, but in the opposite direction to that intended. The aileron action is said to be *reversed.*

Some of the earliest researchers to analyze the control surface effectiveness problem were those at the Royal Aeronautical Establishment in Farnborough, England.[16,17, 18]

1.4 - Swept wing aeroelasticity

In 1935 Dr. Adolph Busemann and colleagues in Germany proposed sweeping wings to delay the onset of wave drag due to high speed flow compressibility near Mach 1.[19] High speed swept wing aircraft designs appeared in Germany late in World War II. Some were swept forward while others

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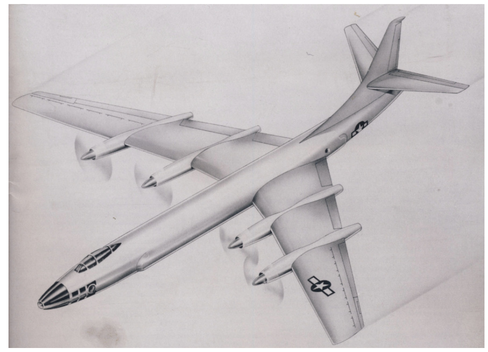

Figure 1.18 – The swept wings provided many aeroelastic analysis challenges after World War II: left, B-47 jet bomber; right, proposed Convair B-50 long range bomber (Ref. 20)

were swept back. There are three reasons to sweep a wing forward or backward: 1) to improve longitudinal stability by reducing the distance between the aircraft center of gravity and the wing aerodynamic center; 2) to provide longitudinal and directional stability for tailless (flying) wings; 3) to delay transonic drag rise (compressibility).

Several swept wing designs were seriously considered for long range, high-speed bombers immediately following World War II. The forward swept wing design was developed first in Germany (the Ju-284); several serious proposals for long range bombers were developed using this configuration. One such configuration is shown in Figure 1.18.[20] The B-47 jet bomber, also shown in Figure 1.18, was built as the successor to the B-36. The B-47 was the first to encounter and to address high speed swept wing aircraft aeroelastic issues ranging from control effectiveness to flutter. The B-47's high aspect ratio swept wings and speed of 607 mph also led to more effective ways of analyzing aeroelastic behavior.[21] Aeroelastic issues for both of these designs are discussed in Chapter 3.

Figure 1.19 shows two different aerodynamic load distributions along a 35 degree sweptback wing, one for a hypothetical rigid wing, the other for an identical, but moderately flexible, wing. The total lift, shown as the areas under each of the two curves, is equal; the aircraft angle of attack for the flexible surface must be larger than the rigid surface.

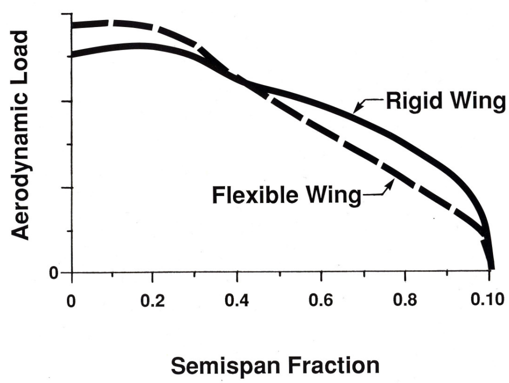

Figure 1.19 –Typical sweptback wing lift re-distribution

As shown in Figure 1.20, sweptback wing bending displacement reduces wing section angles of attack, leading to three static aeroelastic problems: 1) flexible sweptback wings are lift ineffective because wing bending reduces the total wing lift for a given wing angle of attack; 2) bending deformation moves the wing center of pressure inboard and forward; and, 3) bending displacement reduces sweptback wing aileron effectiveness as well as sweptback tail/rudder effectiveness so much that ailerons are often replaced by spoilers. On the positive side, local angle of attack reduction created by swept wing bending counters the increased angle of attack created by torsion. This

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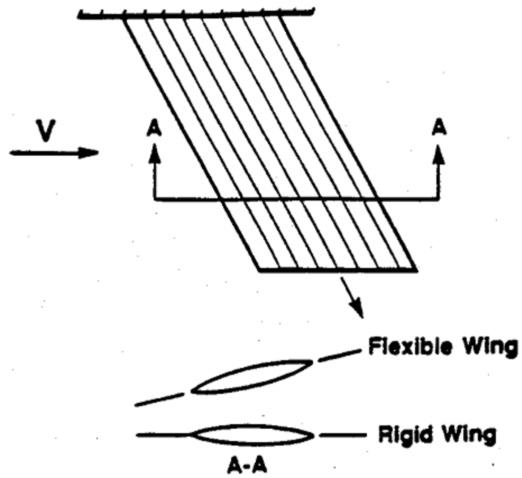

Figure 1.20 – Swept wing bending flexibility produces wash-out that reduces local angle of attack at wing sections such as A-A.

cancellation makes wing divergence unlikely if wings are swept back more than 10-15 degrees, but more likely if the wings are swept forward. The same issues can be seen if the flow direction is reversed in Figure 1.20 to create a forward swept wing. In this case bending increases the angle of attack and exacerbates the tendency of the wing to develop lift, leading to low divergence speeds and abandonment of this particular configuration.

The increased complexity of high-speed swept wing jet aircraft construction challenged engineers and analysts.[22] Swept wing static aeroelasticity studies appeared in the late 1940's and early 1950's.[23, 24, 25, 26] Static aeroelastic methods for the complete airplane were developed at the Boeing Company about 1950 and published in an NACA report.[27]

One of the primary results of swept wing aeroelasticity was the need for inclusion of aeroelastic effects in the assessment of flight mechanics. An NACA report by Skoog[28] describes the off-setting effects of changes in the wing aerodynamic center due to flexibility and the effect of reduced lift-curve slope (reduced lift effectiveness).

Sweeping back a wing or a tail surface exacerbates the control reversal problem created by torsional flexibility. The incremental lift created by a downward aileron or elevator deflection not only causes detrimental nose-down twist of an aft swept surface, but also bends the surface upward. Upward bending of a sweptback wing amplifies the aileron effectiveness problem. As a result, the spanwise location and sizing of control surfaces on sweptback wings is crucial to the success of the design. In some cases, the use of ailerons at high speeds is abandoned altogether and lift spoiler devices used in their place.

Design, testing and certification created new demands for analytical fidelity. In the case of the swept wing the complexity of the structure drove aeroelastic development. This led Boeing engineers to develop a new aero/structural analysis method called the *finite element method*. The 1956 paper by Turner, Clough, Martin and Topp,[29] coupled with the emergence of high speed computers, laid the groundwork for a new powerful method of structural analysis that has since been embraced by the entire structural engineering community.

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1.5 - High-speed and special flutter problems

In 1947 the Bell X-1 broke the sound barrier; soon many fighters and a few bombers were flying at supersonic speeds. In 1958 the X-15, shown in Figure 1.21, flew at hypersonic speeds near the edge of the Earth's atmosphere. By 1982 the first Space Shuttle showed that it could operate at speeds ranging from subsonic to hypersonic.

High speed flight created new aeroelastic challenges for launch vehicle structures. The X-15 was the first airplane designed and flight tested with a high temperature materials and a structure designed to operate in a high-temperature environment. A new aeroelastic instability, called *panel flutter*, was observed on the X-15 tail and side fairings. These panel locations are shown in Figure 1.22.[30]

Supersonic panel flutter occurs because of modal coupling between the first two bending modes of a plate-like surface. Panel flutter differs from other wing flutter because; 1) only one side of the plate-like structure is exposed to high speed flow while the other is in contact with dead air; 2) its onset usually leads to limit cycle oscillations that create a severe, sustained, acoustic environment and lead to high cycle fatigue. Early models of the German V-2 rocket were rumored to have been lost due to flutter of panels near the rocket nose.

High-speed flutter instabilities also include single degree of freedom flutter.[31] Unlike classical flutter requiring modal interaction, single degree of freedom flutter involves instability mechanisms such as loss of damping, flow separation, pure pitching oscillations or pure bending oscillations. Single degree of freedom pitch oscillations require oscillation at special reduced frequencies and pitch axis locations.

Figure 1.21 – The North American X-15 first flew at hypersonic speeds in 1958

One common limit cycle flutter with one degree of freedom is *control surface buzz*. This instability requires shock-wave/boundary layer interaction to trigger separated flow with periodic shock wave reattachment. This instability is highly nonlinear with the onset of a limit cycle oscillation dependent upon unsteady aerodynamic phenomena.

AREAS AFFECTED BY PANEL FLUTTER

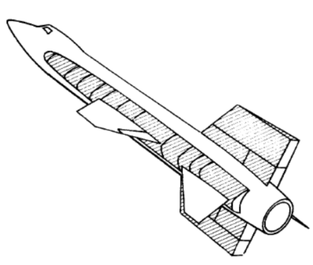

Figure 1.22 – X-15 showing fuselage and vertical tail areas affected by panel flutter.

During the post-World War II era, airplane size and speeds increased. A variety of different configurations, including flying wings, appeared. Some of these airplanes had short-period modes that coupled with flexible vibration modes such as wing bending. As a result, a new aeroelastic phenomenon called *Body Freedom Flutter* appeared.

Body freedom flutter occurs as the aircraft short period mode frequency increases with airspeed and comes in close proximity to a wing vibration mode, usually wing bending. The modal coupling between aircraft pitching motion and wing structural vibration appeared on the

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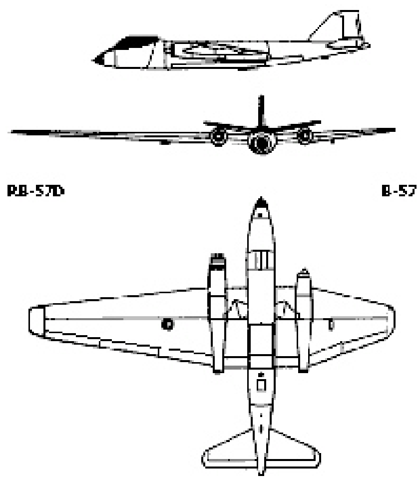

Figure 1.23 – Composite diagram of the RB-57 showing greatly lengthened wing (left), compared to the B-57 wing (right).

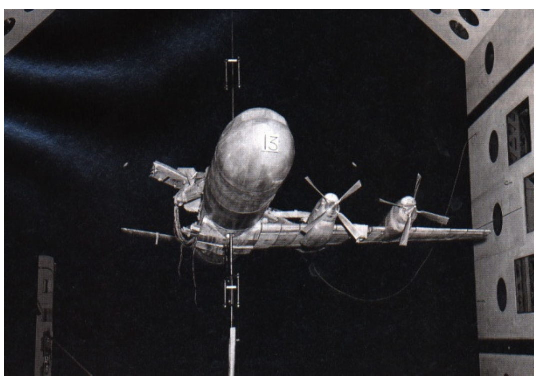

Figure 1.24 – Lockheed Electra after wind tunnel flutter failure

RB-57 reconnaissance aircraft, a version of the B-57 with extended, high aspect ratio wings and a short fuselage, shown in Figure 1.23. Body-freedom flutter is also a possible mode of instability on flying wings and forward swept wing designs. We will discuss this in Chapter 4.

Engine/airframe flutter appeared in the early 1960's. The victim was the Lockheed *Electra* shown in Figure 1.24. In the fall of 1958, Lockheed began deliveries of a new four-engine turboprop civil transport to U.S. airlines. Soon after introduction to the civil transport fleet, the Electra suffered two widely publicized fatal accidents. On September 29, 1959, an Electra cruising near Buffalo, Texas disintegrated in the air. Investigations revealed that the left wing had separated from the aircraft in flight. On March 17, 1960, an Electra crashed near Tell City, Indiana. Its right wing was found over 11,000 ft from the crash site, torn from the aircraft at high altitude.

Lockheed and NASA/Langley engineers identified a dynamic phenomenon called *propeller-whirl flutter* as the probable cause. Propeller-whirl flutter occurs when the dynamic oscillation of the engine mounts interacts with the gyroscopic torques produced by the engine/propeller combination. This interaction results in an unstable wobbling motion that quickly causes catastrophic flutter of the wing. This unstable oscillation can tear the aircraft apart in 30 seconds. The engine mounts on Electra aircraft were strengthened (and stiffened). The Electra and its derivative, the Navy P-3 anti-submarine patrol aircraft operated without further accidents.

1.6 - Controlling and exploiting aeroelasticity – aeroelastic tailoring and aeroservoelasticity

In 1958, only a little more than fifty years after the Wright Brothers first flew at Kitty Hawk, the U.S. Air Force began design studies on a supersonic bomber. The winner of the design competition was North American Aviation and the bomber was the B-70, shown in Figures 1.25 and 1.26.

The XB-70 was a large delta-wing aircraft with a taxi weight of 542,000 pounds and a top speed exceeding Mach 3 above 70,000 feet. It was 189 feet long. The XB-70 was the first supersonic morphing aircraft. As indicated in Figure 1.25 the XB-70 employed a movable canard surface with

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trailing edge flaps, a variable incidence nose for visibility during landing and take-off to reduce drag at high speed, and wing tips with a fold at the 40% span position. These tips were folded downward at transonic and supersonic speeds to increase lift to drag and improve lateral control. The maximum fold angle was 62.5 degrees; the fold mechanism consisted of six electric actuators on each wing portion.[32]

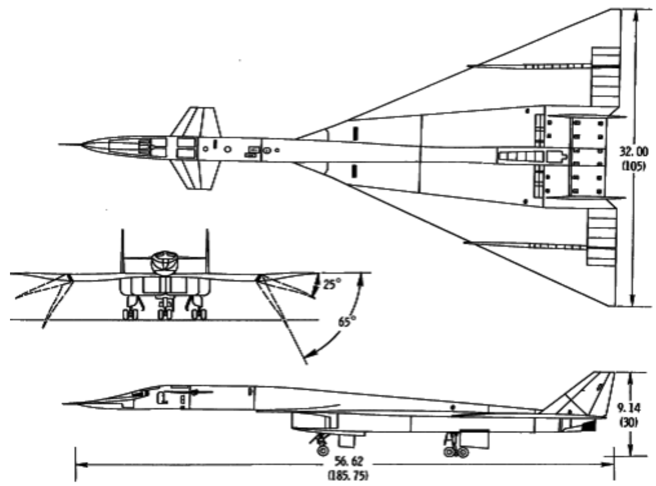

Figure 1.25 – The XB-70; the first supersonic morphing aircraft used a variable incidence drooping nose and folding wingtips controlled in-flight by a total of twelve actuators.

Although few of the original XB-70 structural analyses survive today, the engineering problems are obvious. The fuselage forward of the XB-70 delta wing is flexible, creating a great deal of cockpit vibration in turbulent air. Accurate aeroelastic modeling was difficult given the state-of-the-art in the late 1950's, but important, given the Mach 3 speed (2000 mph) and the thin wing design.[33]

Figure 1.26 – An XB-70 takes flight at NASA Dryden Research Center, California

A stability augmentation system called FACS (Flight Augmentation Control system) was designed and implemented. A "modal model" was used to describe the structural dynamics. Modal modeling will be discussed in Chapter 4.

With the advent of modern advanced composite materials, such as graphite/epoxy and boron/aluminum, passive aeroelastic control, known as *aeroelastic tailoring*, has allowed including structural stiffness design to control deformation mode coupling. Strong directional stiffness exhibited by advanced composite materials such as graphite/epoxy provides the ability to decouple or couple aeroelastic deformations. Aeroelastic tailoring through use of composite laminate design is used to increase lift effectiveness, control effectiveness and flutter speed. We will consider this subject in Chapter 3.

Figure 1.27 - The DARPA X-29 research aircraft promoted aeroelastically tailored advanced composite materials to control undesirable static aeroelastic effects.

In the early 1970's aeroelastic tailoring was applied to sweptforward wing designs with inherently low wing

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divergence speeds. The DARPA X-29 research aircraft shown in Figure 1.27 was the result of these efforts.

Aeroelastic tailoring introduces structural bending/torsion elastic coupling by rotating the laminate fiber direction off-axis of the wing sweep axis. Sweptforward wings with the fibers aligned along the wing swept axis leads to deformation called *wash-in* and increased airloads. This reduces the wing divergence airspeed compared to an unswept wing. Substantial added structural stiffness (and weight) is required to provide aeroelastic stability.

Orienting laminate fibers slightly off-axis changes bend/twist displacement coupling. As the wing bends upward it twists in the nose-down direction, creating *wash-out*. This reduces the local airloads and increases wing divergence speed without extra weight. This concept will be discussed in Chapter 3.

Aeroelastic phenomena such as aileron reversal have been exploited to create actively controlled, light-weight structures. The X-53 Active Aeroelastic Wing (AAW) test aircraft shown in Figure 1.28 uses a combination of active and passive aeroelastic control to produce a highly flexible, light-weight control system.[34] The X-53 wing has a relatively low reversal speed; it uses ailerons to create wing distortion much like wing warping to roll the aircraft.

Figure 1.28 - X-53 Active flexible wing aircraft

Aeroservoelasticity uses interactive, active flight control to modify aeroelastic dynamic response and stability. In the past few decades, aircraft active flight control has brought flight mechanics much closer to aeroelasticity than it has been in the past. Until a few decades ago, except in unusual cases, aeroelasticians isolated lifting surface aeroelastic response from vehicle response. Aeroservoelasticity began by improving XB-70 supersonic bomber ride quality. Later active flutter suppression using actively controlled ailerons was demonstrated on a B-52 test aircraft.[35]

1.7 - Lessons of a century of aeroelastic events

The history presented in Chapter 1 is but a small part of the history that has shaped aeroelasticity over the last century. Today, aeroelasticity is regarded as a mature discipline. However, this maturity does not mean that aeroelasticity does not face challenges.

Figure 1.29 shows a Venn diagram[36] similar to that shown in Figure 1.1, but with far more complexity, indicative of the areas that a century of progress have allowed or required aeroelasticity to encompass. More areas have become interactive as speeds increase and missions change. Analytical methods, which we will discuss in the ensuing chapters, have also responded to provide analytical model fidelity.

More importantly, aeroelasticity has moved into the mainstream of aircraft development. Many aircraft certification criteria involving aeroelastic requirements and constraints apply to today's aircraft. These criteria are the product of one hundred years of experience, innovations and mistakes.

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In the United States, the Federal Aviation Administration regulations govern certification of commercial aircraft.[37] In particular, FAR 23.369 and FAR 25.369 require definition of the transport aircraft flutter speed and assurances that it is 20% above the limit dive speed while FAR 23.301(c) and FAR 25.301(c) require that the aircraft loads properly account for deflection of the structure. Substantial portions of aircraft development budgets are set aside for analysis and scaled aeroelastic models for wind tunnel testing to certify that aeroelastic phenomena do not interfere with flight performance or mission goals. Additional time is appropriated for full-scale flight testing and certification.

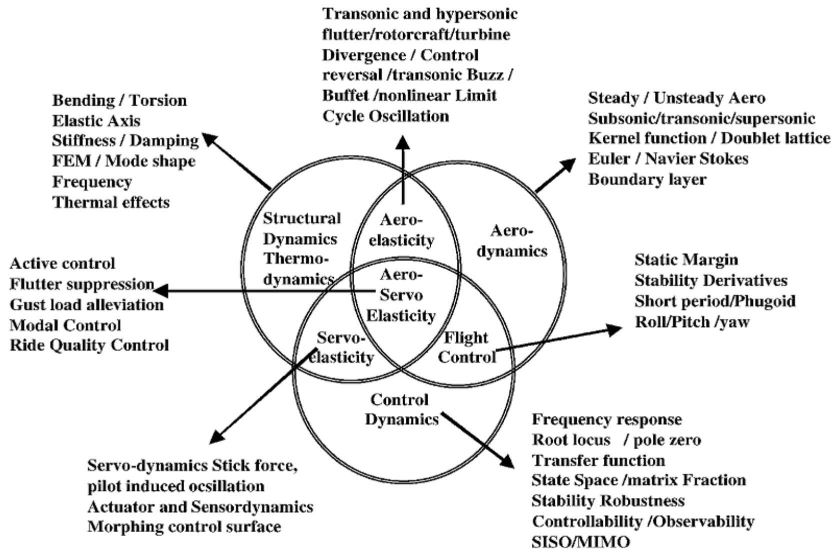

Figure 1.29 – A modern Venn diagram showing modern coupled aeroelastic disciplines (Ref. 36)

Aeroelasticians must be knowledgeable in the areas shown in Figures 1.1 and 1.29 and masters of the art of interconnecting these areas. More importantly, they must know how to interpret analytical results to connect these results to the real world of engineering. While the availability of large computer codes such as NASTRAN has made the analyst's job easier, it goes without saying that a sixth-grader with an intimate knowledge of NASTRAN data preparation is not an aeroelastician. Very often, in our desire to calculate numbers using elegant models and even more elegant mathematics and symbols, we miss the ability to see what is going on and thus avoid a problem before it happens or, when faced with two or three alternatives, fail to quickly grasp the best or the worst of the choices.

That bit of philosophy brings us to the purpose of the next chapters. Winston Churchill said, in November 1942, commenting on British and American victories in North Africa, said "Now this is not the end. It is not even the beginning of the end. But it is, perhaps, the end of the beginning." That is the philosophy that guided the preparation of the next three chapters. At the end of these chapters you will not be an aeroelastician, but you will be ready to learn to be an aeroelastician.

Chapter 2 introduces basic terminology used in aerodynamics and structural analysis. A simple analytical model is developed to understand the scope and causes of the static aeroelastic interactions. The scope includes the important influence of sweep on static aeroelastic behavior. Chapter 3 also has a discussion of the effects of advanced composite material layout, aeroelastic tailoring, aeroelastic phenomena.

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Chapter 4 examines dynamic aeroelastic phenomena, particularly flutter. This chapter begins by defining terminology and analyzing the effects of aerodynamic forces on free vibration of wings and other lifting surfaces. The features of unsteady lift produced by oscillating surfaces are then explained and examined. Several methods of solving for flutter speed are then described.

Two MATLAB programs are included with Chapters 3 and 4. The first MATLAB code allows a student to solve for aeroelastic effects such as roll effectiveness and lift distribution for finite wings whose structural behavior is modeled as beam-like. The second MATLAB program allows students to solve for flutter speeds for typical section airfoils and to use control laws to modify these speeds.

1.8 - Textbooks and references

Over the past thirty years at Purdue nearly one thousand students have used some version of this text to begin their study of aeroelasticity. Some have even become aeroelasticians. On the other hand, there are several aeroelasticity textbooks, classical and modern, that describe aeroelastic phenomena in all flight speed regimes and provide methods to analyze them. Many of these methods have been supplanted by modern computational developments, but these textbooks are an excellent source of fundamental explanations, classic examples and valuable, basic computational techniques that can be used for first estimates of aeroelastic effects. The following references are recommended for further study. My favorites are references 1 and 3.

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